Supplement to "New General Analytic and Numerical Methods in Constrained Optimization with Applications to Optimal Consumption" by Dean Banerjee and John Gregory

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ABSTRACT This paper is intended as a supplement which points out one error found and also works out a practical example involving optimal spending. This data was all generated with an excel spreadsheet.

I. Corrections

The constant in the formula for optimal spending was miscalculated. This can be calculated by plugging the solution into the integral constraint and solving for the constant. The correct expression for the spending rate is shown below:

$$\dot{s}(t) = \frac{E \bullet b \bullet a}{(1-a) \bullet exp\left(\frac{a \bullet b \bullet L}{1-a} - 1\right)} \bullet exp\left(\frac{b \bullet t}{1-a}\right)$$

II. Specific Example

In this section an example will be given and calculated with an excel spreadsheet for a spending period of 25 years. This will be done in several steps as outlined below:

A. Calculation of α
B. Calculation of future value of earnings and assets.
C. Calculation of optimal spending rate.

A. Calculation of α

Recall that α gives an indication for how much utility a dollar gives. A sample chart relating the spending rate and utility gained is shown below. Note that this chart is specific to each individual.

Spending Rate/Year	Utility Gained from Spending This
\$10, 000	10.0
\$20, 000	10.8
\$50, 000	11.8
\$100, 000	12.6
\$150, 000	13.2
\$200, 000	13.6
\$300, 000	14.1
\$500, 000	14.9

So, the utility, U, as a function of spending, s' is approximated by $U \approx 4 \bullet \dot{s}^{0.1}$

So $\alpha = 0.1$ for this example.

B. Calculation of All Future Earnings

For this, it is important to take into account interest rate, inflation, assets at the beginning of the time period, and assets that are to be leftover. The summary information is given below:

Attribute	Value	Comments
Salary	Future Value = $ 3.062 M	Salary of $30,000 with an annual 5% raise
Starting Capitol	Future Value = $65.13 K	Starting Capitol of $12 K at time = 0
Future Value of All Earnings	$ 3.122 M	Salary + Starting Capitol
Money to be Leftover	$ 1.570	Future Value
Interest Rate	10 %	Excludes Inflation
Inflation Rate	3 %	
α	0.1	Calculated Previously
β	0.068	Includes Inflation
Lifetime	25 Years	

C. Calculation of Optimal Spending Rate

Year	Present Value Spent	Future Value Spent	Utils Gained
1	$ 10,799	$ 56,647	10.1
2	$ 11,642	$ 57,075	10.2
3	$ 12,551	$ 57,505	10.2
4	$ 13,531	$ 57,939	10.3
5	$ 14,587	$ 58,376	10.3
6	$ 15,726	$ 58,817	10.4
7	$ 16,954	$ 59,261	10.4
8	$ 18,278	$ 59,708	10.5
9	$ 19,705	$ 60,158	10.5
10	$ 21,243	$ 60,612	10.5
11	$ 22,901	$ 61,070	10.6
12	$ 24,689	$ 61,531	10.6
13	$ 26,617	$ 61,995	10.7
14	$ 28,695	$ 62,463	10.7
15	$ 30,936	$ 62,934	10.8
16	$ 33,351	$ 63,409	10.8
17	$ 35,955	$ 63,887	10.9
18	$ 38,762	$ 64,370	10.9
19	$ 41,788	$ 64,855	11.0
20	$ 45,051	$ 65,345	11.0
21	$ 48,568	$ 65,838	11.1
22	$ 52,360	$ 66,335	11.1
23	$ 56,448	$ 66,835	11.2
24	$ 60,855	$ 67,339	11.2
25	$ 65,606	$ 67,848	11.3
Σ	$ 492,381	$ 1,552,152	267.5

III. Concluding Remarks

This example shows how to apply the optimal spending model to a specific example. Note that the results are very dependent on the interest rates. For higher interest rates and higher values for α, there is a tendency for this model to suggest hoarding money until the end. This model could be used to model many different scenarios including hyperinflation or starting out in debt.